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Math 315, Section 2
Exam 2
Instructor: David G. Wright
5-7 March 2009

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1. (30%) Complete the following definitions:

   (a) A set U in $\mathbb{R}$ is *open* if

   (b) A point p is a *limit point* of a set A if

   (c) Let $f : A \to \mathbb{R}$ be a function. Then $\lim_{x \to c} f(x) = L$ means

   (d) Let $f : A \to \mathbb{R}$ be a function. Then f is *continuous* at $c \in A$ means

   (e) A set C in $\mathbb{R}$ is *compact* if

   (f) Let $f : A \to \mathbb{R}$ be a function. Then f is *uniformly continuous* means

2. (10%) If U and V are open sets, prove $U \cap V$ is also open.

3. (10%) A set I is an *interval* means that if $x, y, \in I$, then everything between x and y is also in I. Prove that an interval is connected.

4. (10%) Suppose $f, g : \mathbb{R} \to \mathbb{R}$ are continuous functions. Show that the composition $g \circ f(x) = g(f(x))$ is continuous.

5. (10%) Prove that a continuous function on a compact set has a maximum; i.e., if $f : K \to \mathbb{R}$ is a continuous function and K is compact, then there is a $c \in K$ so that $f(x) \leq f(c)$ for all $x \in K$.

6. (10%) Let $f(x) = \dfrac{1}{x^2}$. Show

   (a) f is uniformly continuous on $[1, \infty)$

   (b) f is not uniformly continuous on $(0, 1]$.

7. (10%) Let f be a continuous function on the closed interval $[0, 1]$ with range also contained in $[0, 1]$. Prove that f must have a fixed point; that is, show $f(x) = x$ for at least one value of $x \in [0, 1]$.

8. (10%) Let A be a subset of $\mathbb{R}$ and $\overline{A}$ be A along with all the limit points of A. Show that $\overline{A}$ is a closed set.